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Heterotic/Type II Duality in D=4 and String-String Duality[◇]

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ABSTRACT

We discuss the structure of Heterotic/Type II duality in four dimensions as a consequence of string-string duality in six dimensions. We emphasize the new features in four dimensions which go beyond the six dimensional vacuum structure and pertain to the way particular K3 fibers can be embedded in Calabi-Yau threefolds. Our focus is on hypersurfaces as well as complete intersections of codimension two which arise via conifold transitions.

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We discuss the structure of Heterotic/Type II duality in four dimensions as a consequence of string-string duality in six dimensions. We emphasize the new features in four dimensions which go beyond the six dimensional vacuum structure and pertain to the way particular K3 fibers can be embedded in Calabi-Yau threefolds. Our focus is on hypersurfaces as well as complete intersections of codimension two which arise via conifold transitions.

1. Introduction

Over the last two years a large variety of duality conjectures has been put forward, relating different compactifications of the basic five string theories. It clearly would be helpful if all these dualities [1–8] could be traced back to a few basic ancestors. Progress in this direction has been made in several articles [9]. In the present paper we focus on the Heterotic/Type II duality in D=4 [3] as a consequence of string-string duality in D=6. The expectation that such a relation between dualities might hold is based on the recognition [4] that this duality in D=4 is possible for K3-fibered Calabi-Yau threefolds. Thus one might expect to be able to apply string-string duality fiber-wise in the limit when the fiber varies little - the adiabatic limit of [5]. The concrete implementation of this idea [10], which we will describe in the following, shows that there are a number of new twists that introduce additional structure beyond that of the K3 in D=6.

2. String-String Duality in D=6

It was observed in [11] that the heterotic string compactified on the four-torus T^4 has the same moduli space $\mathrm{SO}(20,4;\mathbb{Z})\backslash\mathrm{SO}(20,4;\mathbb{R})/\mathrm{SO}(20;\mathbb{R})\times\mathrm{SO}(4;\mathbb{R})$ as the type IIA string compactified on the K3 surface. Thus the compactification of two different string theories on the only two Calabi-Yau complex surfaces turn out to be identical. Dualities in 4D then follow by compactifying these vacua further on 2-tori [1,12].

3. Heterotic/Type II Duality in D=4

Evidence for a more general dualities in four dimensions has been presented in [3]. It was shown there that Calabi-Yau threefolds exist for which one can find bundles V_i on $\mathrm{K3}\times\mathrm{T}^2$ such that after Higgsing one finds the relation

$$\mathrm{Het}(\mathrm{K3}\times\mathrm{T}^2)\longleftrightarrow\mathrm{IIA}(\mathrm{CY}_3). \qquad (1)$$

With hindsight such a relation might have been expected for a particular class of Calabi-Yau threefolds. By multiplying $\mathrm{T}^4\longrightarrow\mathrm{T}^4\times\mathbb{P}_1$ and K3 $\longrightarrow\mathrm{K3}\times\mathbb{P}_1$ and viewing this as the *local* structure of the threefolds, which has to be twisted such as to make these into fibered Calabi-Yau manifolds, one might hope to establish this duality in four dimensions.

A concrete procedure to do this twisting has been described in [10]. This construction shows to what extent it is in fact the K3 structure which is responsible for the duality in 4D. It also explicates the new features that arise when string-string duality is lifted to from D=6 to D=4. We now review some of the salient properties of the

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relevant twist map.

3.1. The Twist Map

The starting point of the construction of [10] is a K3 surface with an automorphism $\mathbb{Z}_\ell \ni \mathbf{m}_\ell$. Depending on the order of this cyclic symmetry group we choose a Riemann surface $\mathcal{C}_\ell$ of genus $g(\ell)$ and the projection $\pi_\ell : \mathcal{C}_\ell \longrightarrow \mathbb{P}_1$. The twist map

$$\mathcal{C}_\ell \times K3 \Big/ \mathbb{Z}_l \ni \pi_\ell \times \mathbf{m}_\ell \longrightarrow \mathrm{CY}_3 \tag{2}$$

then produces the explicit twisting.

For the class of weighted Calabi-Yau hypersurfaces this map takes the following explicit form. Given a K3 surface $\mathbb{P}_{(k_0,k_1,k_2,k_3)}[k]$ with $k/k_0 = \ell \in \mathbb{N}$ we define the curve $\mathcal{C}_\ell = \mathbb{P}_{(2,1,1)}[2\ell]$ of genus $g(\ell) = (\ell-1)^2$ and the map

$$\mathbb{P}_{(2,1,1)}[2\ell] \times \mathbb{P}_{(k_0,k_1,k_2,k_3)}[k] \Big/ \mathbb{Z}_\ell \\ \longrightarrow \quad \mathbb{P}_{(k_0,k_0,2k_1,2k_2,2k_3)}[2k] \tag{3}$$

via

$$((x_0,x_1,x_2),(y_0,\ldots,y_3)) \longrightarrow \\ \left(x_1\sqrt{\frac{y_0}{x_0}}, x_2\sqrt{\frac{y_0}{x_0}}, y_1, y_2, y_3\right). \tag{4}$$

This map then embeds the orbifold of the product on the lhs into the weighted four-spaces $\mathbb{P}_{(k_0,k_0,2k_1,2k_2,2k_3)}$ as a hypersurface of degree $2k$.

3.2. Properties of the Twist Map

The map (4) shows that the structure of the threefold is indeed determined by a single K3 surface, a feature that supports the fiber-wise reduction of D=4 duality. Using the results of [2] it is in particular to be expected that the degeneration structure of the K3 surface will play an important role in the determination of the gauge structure of the heterotic dual.

The map also shows, however, the new features introduced by the twisting: the action of $\pi_\ell \times \mathbf{m}_\ell$ has fixed points which have to be resolved. This resolution introduces new cohomology and therefore the heterotic gauge structure is not completely determined by the K3 fiber.

In the weighted category this aspect has two manifestations:

1. If $k_0 = 1$, as was the case in the original considerations of [4,6,8,7], the action of $\mathbb{Z}_\ell$ generates a singular curve on the threefold which lives in the K3 fiber.

   This curve, which is not present in the original K3 surface, has the effect of introducing additional branchings in the resolution diagram of the Calabi-Yau threefold. It is this branching which determines the final gauge structure of the heterotic dual of the IIA theory on the threefold.

2. More generally one encounters $k_0 > 1$, considered for threefolds in [13] and for fourfolds in [14]. In such a situation the orbifolding $\mathbb{Z}_\ell$ generates further singularities on the threefold (and the fourfold). Depending on the structure of the weights these additional singularities can be either points or additional singular curves (and surfaces for fourfolds).

As an example which illustrates the first point we consider the K3 surface

$$K = \{y_1^{42} + y_2^7 + y_3^3 + y_4^2 = 0\} \in \mathbb{P}_{(1,6,14,21)}[42]. \tag{5}$$

K has an automorphism $\mathbb{Z}_{42}: (y_1,y_2,y_3,y_4) \mapsto (\alpha y_1, y_2, y_3, y_4)$, where α is a 42^{nd} root of unity. Thus the associated curve $\mathcal{C}_{42} = \mathbb{P}_{(2,1,1)}[84]$ of the pair (K3, $\mathbb{Z}_{42}$) is a degree 42 cover of $\mathbb{P}_1$, branched at the 84 roots of -1 under the projection $\pi_{42} : \mathbb{P}_{(2,1,1)}[84] \longrightarrow \mathbb{P}_1$, the map π_{42} is given explicitly by $\pi_{42}(x_0,x_1,x_2) \mapsto (x_1,x_2)$. The action of $\mathbb{Z}_{42}$ on the product then has 84 fixed divisors, namely the copies of K lying over the points $q_1,\ldots q_{84} \in \mathcal{C}_{42}$ which is the fixed point set of $\mathbb{Z}_{42}$ acting on $\mathcal{C}_{42}$. These fixed divisors are the degenerate fibers of the K3 fibration of the quotient $\mathbb{Z}_{42}\backslash\mathcal{C}_{42} \times K$ over $\mathbb{P}^1$ (given by projection to the first factor). Instead of resolving the singularities of the quotient, we use the twist map (4). The image is the well-known Calabi-Yau $M \in \mathbb{P}_{(1,1,12,28,42)}[84]$. The degenerate fibers of M as a K3 fibration are cones over the curve $C = \{y_2^7 + y_3^3 + y_4^2 = 0\}$ in $\mathbb{P}_{(6,14,21)}[42]$. After resolution of the ambient projective space, the Euler number is $\chi(C) = 11$. There is a $\mathbb{Z}_2$, a $\mathbb{Z}_3$ and a $\mathbb{Z}_7$ fixed point on the curve, leading to 1,

2 and 6 new curves, respectively. Thus we have $h^{1,1}(M) = 2 + 9 = 11$ and the Euler number is obtained from the fibration formula [10]

$$\chi(M) = 2(1-\ell)\cdot 24 + 2\ell(\chi(C) + 2k_1 - 1) \qquad (6)$$

as $\chi = -960$. Finally, to find the invariant part under the monodromy, we only have to determine the Picard lattice of K. Our automorphism group $\mathbb{Z}_{42}$ is the group denoted H_K there, and this group leaves precisely the Picard lattice invariant. Now we apply the results of Kondo [15] which imply that K is the *unique* K3 surface with a $\mathbb{Z}_{42}$-automorphism, and that K is an elliptic surface (5) for which Kondo shows that the Picard lattice is $S_K = U \oplus E_8$. Thus this is the lattice of the gauge group of the heterotic dual. In terms of the curve C this invariant lattice is described by the resolution diagram above, while in terms of the elliptic surface it is a union of a section and a singular fiber of type $\mathbf{II}^*$ (see [10] for more details).

The twist map (4) furthermore makes explicit how a single K3 surface can lead to different threefolds: consider the K3 surface $\mathbb{P}_{(1,2,3,3)}[9] \ni \{y_0^9 + y_1^3 y_2 + y_2^3 + y_3^3 = 0\}$ with the automorphism $\mathbb{Z}_9 : (y_0, y_1, y_2, y_3) \longmapsto (\alpha y_0, y_1, y_2, y_3)$. Associated to this pair (K3, $\mathbb{Z}_9$) we choose the curve $\mathbb{P}_{(2,1,1)}[18]$ of genus $g = 17^2$. With these ingredients the twist map becomes

$$\mathbb{P}_{(2,1,1)}[18] \times \mathbb{P}_{(1,2,3,3)}[9]/\mathbb{Z}_9 \longrightarrow \mathbb{P}_{(1,1,4,6,6)}[18] \qquad (7)$$

leading to a weighted hypersurface with Hodge numbers $(h^{(1,1)}, h^{(2,1)}) = (9, 111)$. Using the same K3 surface $\mathbb{P}_{(1,2,3,3)}[9]$ of the previous example but now with a different automorphism $\mathbb{Z}_3 : (y_0, y_1, y_2, y_3) \longmapsto (y_0, y_1, y_2, \alpha y_3)$ we are led instead to a different twist map

$$\mathbb{P}_{(2,1,1)}[6] \times \mathbb{P}_{(3,1,2,3)}[9]/\mathbb{Z}_3 \longrightarrow \mathbb{P}_{(3,3,2,4,6)}[18], \qquad (8)$$

resulting in one of the K3 fibrations of [13] with spectrum $(h^{(1,1)}, h^{(2,1)}) = (5, 53)$. Beyond the singular $\mathbb{Z}_2$-curve the twist map in this case also produces a second ($\mathbb{Z}_3-$) singular curve $\mathbb{P}_{(1,1,2)}[6]$ on the threefold.

4. Application to the Unification of Vacua

4.1. Weighted Conifold Transitions

In [16] it was shown how conifold transitions, between Calabi-Yau manifolds, introduced in [17], connect physically distinct vacua in type II string theory in a physically sensible way. In [13] such transitions were generalized to the framework of weighted Calabi-Yau manifolds [18]. In general conifold transitions connect K3-fibered manifolds with spaces which are not fibrations. A simple example is the transition from the quasismooth octic $\mathbb{P}_{(1,1,2,2,2)}[8]$ to the quintic $\mathbb{P}_4[5]$. It can be shown, however, that certain types of weighted conifold transitions exist which do connect K3 fibered manifolds [13]. A simple class of such conifold transitions between fibered manifolds is provided by the weighted splits summarized in the diagram

$$\mathbb{P}_{(2l,2l,2m,2k-1,2k-1)}[2(d+l)] \longleftrightarrow \begin{matrix} \mathbb{P}_{(1,1)} \\ \mathbb{P}_{(2l,2l,2m,2k-1,2k-1)} \end{matrix} \begin{bmatrix} 1 & 1 \\ 2l & 2d \end{bmatrix}, \qquad (9)$$

where $d = (2k-1+l+m)$. Here the hypersurfaces, containing the K3 surfaces $\mathbb{P}_{(2k-1,l,l,m)}[2k-1+2l+m]$, split into codimension two manifolds which contain the K3 manifolds

$$\begin{matrix} \mathbb{P}_{(1,1)} \\ \mathbb{P}_{(l,l,m,2k-1)} \end{matrix} \begin{bmatrix} 1 & 1 \\ l & d \end{bmatrix} \qquad (10)$$

of codimension two.

4.2. Twist Map for Split Manifolds

In order to see the detailed fiber structure of the above varieties of codimension two it is useful to generalize the twist map of [10] to complete intersection manifolds. Consider the K3 surfaces of the type (10) and the associated curves $\mathbb{P}_{(2,1,1)}[2d]$ with $d = (l+m+2k-1)$. With these ingredients we can define a generalized twist map

$$\mathbb{P}_{(2,1,1)}[2d] \times \begin{matrix} \mathbb{P}_{(1,1)} \\ \mathbb{P}_{(l,l,m,2k-1)} \end{matrix} \begin{bmatrix} 1 & 1 \\ l & d \end{bmatrix} \longrightarrow \begin{matrix} \mathbb{P}_{(1,1)} \\ \mathbb{P}_{(2l,2l,2k-1,2k-1,2m)} \end{matrix} \begin{bmatrix} 1 & 1 \\ 2l & 2d \end{bmatrix} \qquad (11)$$

via

$$((x_0, x_1, x_2), (u_0, u_1), (y_0, ..., y_3)) \longrightarrow \left((u_0, u_1), (x_1\sqrt{\frac{y_0}{x_0}}, x_2\sqrt{\frac{y_0}{x_0}}, y_1, y_2, y_3)\right) \qquad (12)$$

Again we see that the quotienting introduces additional singular sets and the remarks of Section 3.2 apply in the present context as well. In particular we see the new singular curve

$$\mathbb{Z}_2: \quad C = \begin{matrix} \mathbb{P}_{(1,1)} \\ \mathbb{P}_{(l,l,m)} \end{matrix} \begin{bmatrix} 1 & 1 \\ l & d \end{bmatrix} \tag{13}$$

which emerges on the threefold image of the twist map.

An example for such a transition between manifolds which are K3-fibered as well as elliptically fibered is given by

$$\mathbb{P}_{(1,1,2,4,4)}[12] \quad \longleftrightarrow \quad \begin{matrix} \mathbb{P}_{(1,1)} \\ \mathbb{P}_{(4,4,1,1,2)} \end{matrix} \begin{bmatrix} 1 & 1 \\ 4 & 8 \end{bmatrix} \tag{14}$$

where the Hodge numbers of the hypersurface are $(h^{(1,1)}, h^{(2,1)}) = (5, 101)$ while those of the codimension two threefold $(h^{(1,1)}, h^{(2,1)}) = (6, 70)$. Here the transverse codimension two variety of the rhs configuration is chosen to be

$$\begin{aligned} p_1 &= x_1 y_1 + x_2 y_2 \\ p_2 &= x_1(y_2^2 + y_4^8 + y_5^4) + x_2(y_1^2 + y_3^8 - y_5^4) \end{aligned} \tag{15}$$

which leads to the determinantal variety in the lhs hypersurface configuration

$$p_{\rm det} = y_1^3 - y_2^3 + (y_1 y_3^8 - y_2 y_4^8) - (y_1 + y_2) y_5^4. \tag{16}$$

This variety is singular at $\mathbb{P}_{(4,4,1,1,2)}[4\ 4\ 8\ 8] = 32$ nodes, which can be resolved by deforming the polynomial.

Contained in these 2 CY–fibrations are the K3 configurations

$$\mathbb{P}_{(2,2,1,1)}[6] \quad \longleftrightarrow \quad \begin{matrix} \mathbb{P}_{(1,1)} \\ \mathbb{P}_{(2,2,1,1)} \end{matrix} \begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix}, \tag{17}$$

where the left-right arrow indicates that these two are indeed related by splitting. The K3 of the rhs is obtained by considering the divisor

$$D_\theta = \{y_4 = \theta y_3\} \tag{18}$$

which leads to

$$\begin{matrix} \mathbb{P}_{(1,1)} \\ \mathbb{P}_{(4,4,1,2)} \end{matrix} \begin{bmatrix} 1 & 1 \\ 4 & 8 \end{bmatrix}, \tag{19}$$

with

$$\begin{aligned} p_1 &= x_1 y_1 + x_2 y_2 \\ p_2 &= x_1(y_2^2 + \theta^8 y_3^8 + y_5^4) + x_2(y_1^2 + y_3^8 - y_5^4). \end{aligned} \tag{20}$$

Because of the weights in the weighted $\mathbb{P}_3$ this is equivalent to the rhs of (17) with

$$\begin{aligned} p_1 &= x_1 y_1 + x_2 y_2 \\ p_2 &= x_1(y_2^2 + \theta^8 y_3^4 + y_5^4) + x_2(y_1^2 + y_3^4 - y_5^4). \end{aligned} \tag{21}$$

The determinantal variety following from this space is

$$p_s = y_1^3 - y_2^3 + (y_1 - \theta^8 y_2) y_3^4 - (y_1 - y_2) y_5^4. \tag{22}$$

But this is precisely what one gets by considering the divisor in the determinantal 3-fold variety (16) and thus we see that the conifold transitions take place in the fiber of the CY 3-fold.

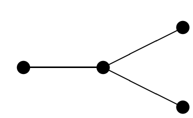
To elucidate the heterotic gauge structure of these K3 fibrations it is necessary to determine the singularity structure of the fibers. In the hypersurface on the lhs the singularities

$$\begin{aligned} \mathbb{Z}_2 &: \quad \mathbb{P}_{(2,2,1)}[6] \\ \mathbb{Z}_4 &: \quad \mathbb{P}_1[4] = 3\text{pts} \end{aligned} \tag{23}$$

are all contained in the K3 fiber. Similarly one finds for the singularities of the codimension 2 space

$$\begin{aligned} \mathbb{Z}_2 &: \quad \begin{matrix} \mathbb{P}_{(1,1)} \\ \mathbb{P}_{(2,2,1)} \end{matrix} \begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix} \\ \mathbb{Z}_4 &: \quad \begin{matrix} \mathbb{P}_1 \\ \mathbb{P}_1 \end{matrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = 3\text{pts} \end{aligned} \tag{24}$$

Both singular curves thus lead to the resolution diagram



which is the Dynkin diagram of SO(8).

The heterotic dual of the type II theory on the hypersurface can be obtained by starting from the torus compactification

$$(M^8 \times T^2, E_8 \times E_8 \times SU(3) \times U(1)^2) \tag{25}$$

with enhanced SU(3) symmetry point on the torus. Compactifying further on a K3 with gauge bundles

$$V_i \longrightarrow K3, \quad i = 1, 2, 3 \tag{26}$$

of rank $2, 2, 3$ respectively with

$$\int_{K3} c_2(V_i) \in \{10, 8, 6\} \qquad (27)$$

respectively and embedding the two SU(2) bundles in the two E_8s respectively and the SU(3) into the SU(3) leads to the gauge group $E_7 \times E_7 \times SU(3) \times U(1)^2$ for the 4–dimensional theory and the numbers

$$N_{\mathbf{56}}(V_1) = 3, \quad N_{\mathbf{56}}(V_2) = 2 \qquad (28)$$

for the two rank 2 bundles as well as the number of singlets

$$N_m(V_1) = 17, \; N_m(V_2) = 13, \; N_m(V_3) = 10, \; (29)$$

which, together with the universal 20 of K3, leads to a total of 60 singlets. The 4D theory thus is described by $(M^4 \times K3 \times T^2, E_7 \times E_7 \times U(1)^2)$ with $(n_H, n_V) = (60, 16)$ and $3 \cdot \mathbf{56}$ in each of the E_7s. Breaking the second E_7 down to SO(8) leads to $(M^4 \times K3 \times T^2, E_7 \times SO(8) \times U(1)^2)$ with $(n_H, n_V) = (67, 13)$ and $3 \cdot \mathbf{56}$ in the surviving E_7. Finally, breaking down the E_7 completely leads to an additional $3 \cdot 56 - 133 = 35$ singlets and therefore one ends up with a model $(M^4 \times K3 \times T^2, SO(8) \times U(1)^2)$ with $(n_H, n_V) = (102, 6)$ and no residual matter. Taking into account the graviphoton and the axion-dilaton multiplet shows that this agrees with the Calabi-Yau 3fold cohomology.

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